

A Computational Analysis of Smart Timing Decisions for Heating Based on an Air-to-Water Heat pump

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Abstract: When renewable energy is locally available for domestic usage, the timing of the usage may be adapted to that. For example, when energy is available in periods when there is sun or wind, it may be most efficient to use this energy in these periods. For air to water heat pumps a similar issue occurs, as their efficiency strongly depends on the outside temperature, which within a day usually makes fluctuations over time. In this paper it is analysed computationally in how far also for a heat pump it makes sense to take these fluctuations into account in its use.

1. Introduction

In the current and future development to the domestic usage of locally available renewable energy as much as possible, an important decision issue concerns the timing of the usage. The availability of renewable energy may strongly depend on the fluctuating natural circumstances including sun, wind and (outdoor) temperature. For example, much energy may be available in periods when there is sun or wind, and therefore it may be economical to decide using this energy in these periods. This also applies to heating based on a heat pump, as it also uses electrical energy (cf. Aste, Adhikari, and Manfren, 2013; Byrne 2013; Chua, Chou, and Yang, 2010; Hepbasli and Kalinci, 2009; Staffell, Brett, Brandonc and Hawkes, 2012; Yang, Shi, and Dong, 2006). For an air to water heat pump, in addition another timing issue occurs, as its efficiency strongly depends on the outside temperature, which within a day can make strong fluctuations over time. Due to this, the performance factor can vary in a range roughly between 2 and 4, which is substantial.

In general there is not much choice in when to take care of heating, since when persons are present (and not sleeping), a certain comfortable temperature will have to be maintained, for example 20°C. However, there is still a choice on which temperature is maintained during the night or on the daytime when nobody is there. Especially for the temperature at night there is often a debate (even for heating based on non-renewable resources such as gas or oil) on whether it should be left free and allowed to become as low as it goes, or whether it should be maintained at a higher level, for example 15°C or 16°C. For resources based on non-

renewable resources such as gas or oil it may be argued that the lower the indoor temperature, the less loss of energy occurs overnight, so it is better not to do any heating in the night. But even in this case of non-renewable resources not everybody is convinced by such an argument; often it is advised by professionals to maintain a night temperature of, for example, 15°C.

For heating by an air to water heat pump there are more serious reasons why this argument by itself cannot be decisive, since the outdoor temperature keeps on decreasing until sunrise, and the lower the outdoor temperature, the lower the efficiency of the heat pump. Much depends on the time at which the heat pump has to do its work in the morning. Theoretically a most economic choice could be when the heating is started on the middle of the day or early afternoon when the outside temperature is the highest. But usually people do not sleep that long. When instead a higher temperature is already needed early in the morning, say at 6:30, then heating may be started around 6, but this is the least efficient timing choice as that is when the outdoor temperature is the lowest of the whole day. It could be conjectured that it may be more economic when a higher temperature is maintained during the night, taking into account that the earlier in the night the higher the outdoor temperature and thus the more efficient the heat pump. This effect might outweigh the loss of energy overnight due to the higher indoor temperature. However, it is not trivial to find out whether this really holds, to which extent, and under which circumstances.

In this paper a computational model is presented that can be used to analyse such timing issues

within a day for an air to water heat pump. Different scenarios can be tried out in simulations based on this computational model. A number of such simulation scenarios have been performed making use of empirical data about the specific temperatures over time during the days in 2012. These experiments, going through all hours in 2012 by steps of half an hour show that overall it is beneficial to let the temperature be a few degrees lower in the night, but lower than that does not provide much benefit. Moreover, the outcomes show that these effects also depend on the time of the year. For example, in the less cold months the temperature at night can be maintained on a higher level without much loss.

2. Theory

The computational approach put forward builds on different ingredients. First, in order to get an overview over a year with its different seasons, data are needed for temperatures over the 24 hours of a day for all days of a year. Mostly available are empirical data about minima and maxima for all days. Therefore some theory is needed that enables to estimate the temperatures over the 24 hours of a day for any day of a year. This will be addressed in Section 2.1. Next a theory is needed on how much heating energy is needed to keep the indoor temperature at a certain level, for a given outdoor temperature. This is addressed in Section 2.2. Finally a theory is needed about the performance of a heat pump depending on the outdoor temperature. This will be addressed in Section 2.3.

2.1. The Outdoor Air Temperature Cycle

Every day the outdoor air temperature follows a cycle with usually a maximum temperature in the afternoon and a minimum temperature at the end of the night. During the day the main driving force of this pattern is the position of the sun, which over time follows a sinusoidal curve. During the night it is different: the air cools down according to a pattern that can be approximated by a negative exponential function of time. Therefore to approximate the daily air temperature cycle, two functions are to be combined: a sinus function for the time interval from sunrise to sunset, and a negative exponential function for the other time interval, from sunset to sunrise. Note that these time intervals also vary over a year. For an example, see Fig. 1; in this case February 21 is shown with maximum temperature

5°C around 3 PM and minimum temperature just below -3°C around 8 AM. In contrast, for example, in September the minimum temperature will be reached between 5 and 6 AM.

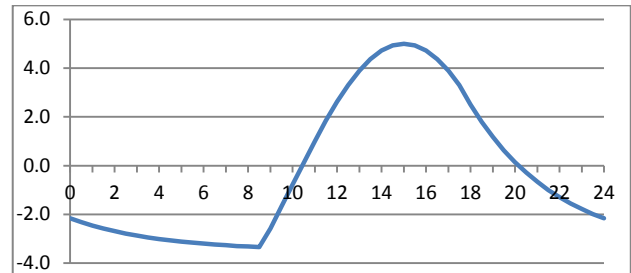


Fig. 1: Example daily temperature cycle: February 21

2.2. Heating Energy Demand

Assuming that the indoor temperature is not always kept at the same level, there are two types of heating energy demands:

- Temperature increase energy demand (*tied*): to increase the indoor temperature
- Temperature maintenance energy demand (*tmed*): to maintain a given indoor temperature

The total energy demand *ed* can be obtained as the sum of these two:

$$ed = tied + tmed$$

The first type of energy demand *tied* relates most to the inside of the house: the heat energetical capacity *C* of the house: this indicates how much energy is needed to raise the temperature by 1 degree. For the second type of energy demand *tmed* it has to be determined how much energy loss a given pair of indoor and outdoor temperatures entails. This relates to the isolation of the border between indoor and outdoor (walls, windows, floor, roof, ventilation). These two demands are addressed separately in Sections 2.2.1 and 2.2.2.

2.2.1. Temperature Increase Demand *tied*

To increase the indoor temperature by ΔT_{id} , for example early in the morning, in addition to the energy *tmed* for maintenance, also extra energy *tied* for temperature increase is needed. The extra energy *tied* is proportional to the temperature difference ΔT_{id} made and relates to the notion of capacity *C* of the house:

$$tied = C \Delta T_{id}$$

2.2.2. Temperature Maintenance Demand *tmed*

To determine how much energy is needed to maintain a certain indoor temperature (which is higher than the outdoor temperature) the concept of *degree day* *dd* is often used. This concept is based on the assumption that for a given time interval Δt the amount of energy needed to maintain a difference in temperature (between indoor and outdoor) is proportional to this difference, and to the length Δt of the time interval (e.g., see Jeong, Kim, and Lee, 2011). The number of degree days for a given time interval Δt directly relates to the difference between the daily outdoor and the indoor temperature T_{id} as follows:

$$dd = (T_{id} - T_{od}) \Delta t \quad \text{when } T_{id} > T_{od} \\ 0 \quad \text{otherwise}$$

Sometimes this is multiplied by a seasonal correction weight factor σ which is 1.1 for the months November, December, January and February, 1 for the months March and October, and 0.8 for the months April, May, June, July, August and September.

The energy needed to maintain a given indoor temperature during such a time interval Δt is the *temperature maintenance energy* *tme* provided to the heating system (*kWh* provided for heating) defined as follows. For each degree day an amount ε of energy (in *kWh*) per day is lost and therefore has to be provided as a compensation. So *tmed* is determined as:

$$tmed = \varepsilon dd$$

Note that these notions also play a central role in the cooling down of the house, when the loss is not compensated by heating. This autonomous cooling down process can be described by a decrease ΔT_{id} in temperature over time interval Δt as follows:

$$\Delta T_{id} = \text{energy loss} / C \\ \text{energy loss} = \varepsilon (T_{id} - T_{od}) \Delta t$$

Therefore cooling down goes according to

$$\Delta T_{id} = (\varepsilon / C) (T_{id} - T_{od}) \Delta t$$

2.3. Performance of a Heat pump

The energy demand as discussed in Section 2.2 is to be provided by a heat pump to the water of the heating system. This energy amount *ep* provided as output to the water of the heating system, is not

equal to the amount *eu* used as input by the heat pump itself, since part of the provided energy *ep* comes from the air in the environment. A central element in this is the seasonal performance factor (*SPF*) which indicates how much electric energy (in *kWh*) is needed (to run the pump) as input to get a certain amount of heating energy as output (in *kWh*) for the heat pump over a certain time period:

$$SPF = \frac{\text{energy output}}{\text{energy input}} = \frac{ep}{eu}$$

For air to water heat pumps this factor usually varies between 2 and 4. For a given water temperature of the heating system, it strongly depends on the outdoor temperature, and in particular the difference between these two temperatures. Manufacturers often only give indications of these performance factors for just a few water and outdoor temperatures, and usually based on laboratory experiments. However, to determine the electricity use of a heat pump over a year, with all its variations in outdoor temperature, it is needed to have a more systematic estimation of *SPF* for a given water temperature and each possible outdoor temperature, in a realistic context. This will be addressed in Section 4. Once both the heating energy demand (see Section 2.2) and the seasonal performance factor is known they can be used to determine the energy usage of the heat pump; for example, for maintaining a given temperature:

$$eu = ep / SPF = ed / SPF = \varepsilon dd / SPF$$

notation	description	unit
<i>SPF</i>	seasonal performance factor	
σ	seasonal weight factor	
T_{od}	average outdoor temperature	°C
T_{id}	average indoor temperature	°C
T_w	water temperature of heating system	°C
<i>dd</i>	degree days	°C day
<i>tmed</i>	heating energy demand for maintaining a temperature	kWh
<i>tied</i>	heating energy demand for increasing a temperature	kWh
<i>ed</i>	total heating energy demand	kWh
<i>eu</i>	heat pump electrical energy use	kWh
ε	energy loss per degree day	kWh/°C day
<i>C</i>	capacity: energy needed per degree increase of temperature	kWh/°C

Tab. 1 Main concepts

3. Related Work

Recently heat pumps have more and more become a subject of research. For a recent overview, see, for example, (Aste, Adhikari, and Manfren, 2013; Byrne 2013; Chua, Chou, and Yang, 2010; Hepbasli and Kalinci, 2009; Staffell, Brett, Brandon and Hawkes, 2012; Yang, Shi, and Dong, 2006). Often the different components of heat pumps are taken as a point of departure and from 'first principles' for their mechanisms properties of the heat pump as a whole are derived (e.g., Byrne 2013). After such derivations the derived properties are to be validated against empirical data. However, a more empirically based perspective can also be taken as a point of departure. In that case overall properties of a heatpump are formulated with certain parameters and based on empirical data the parameters can be estimated; (e.g., Staffell, Brett, Brandon and Hawkes, 2012; Treur, 2013a; Yang, Shi, and Dong, 2006).

The current paper shares the latter perspective in designing and using the computational model. For example, approximation of the dependence of the performance factors is done here in a manner similar to what can be found in (Staffell, Brett, Brandon and Hawkes, 2012; Treur, 2013a; Yang, Shi, and Dong, 2006). An important difference is that here the intraday dynamics are investigated, whereas in the literature as mentioned such small-scale timing issues are not addressed. In order to do so the daily outdoor temperature cycle has to be modelled. For this, inspiration has been found in (Parton and Logan, 1981; Wann, Yah, and Gold, 1985).

4. The Computational Model

The computational model described in this section is based on the theory discussed in Section 2. It has different components for the dynamics of

- the outdoor temperature
- the indoor temperature
- the energy demand
- the energy usage and costs

These will be addressed subsequently in Section 4.1 to 4.4.

4.1. Modelling the Outdoor Temperature

As discussed in Section 2.1 the dynamics of the outdoor temperature can be described by a combination of two functions:

- a sinus function $dot(t)$ (for day time outdoor temperature)
- a negative exponential function $not(t)$ (for night time outdoor temperature)

where t is the time on the day after midnight in hours. These functions can be computed out of empirical data about maximum and minimum day temperatures and of data about the day of the year from which also the sun orbit and the sunrise and sunset times can be determined (using the same model as in (Treur, 2013)). The daytime function $dot(t)$ is defined between sunrise and sunset as follows:

$$dot(t) = b + a \sin((t - 15 + (t_{sunset} - t_{sunrise})/2) \pi / (t_{sunset} - t_{sunrise}))$$

Here t_{sunset} and $t_{sunrise}$ are the times of sunset and sunrise, and

$$a = (T_{max} - T_{min}) / (1 - \sin((t_{sunrise} - 9) \pi / 12))$$

$$b = T_{max} - a$$

with T_{max} and T_{min} the maximum and minimum temperature that day.

The nighttime outdoor temperature function $not(t)$ is defined as follows:

$$not(t) = T_{min} - d + (T_{sunset} - T_{min} + d) e^{-\alpha(t - t_{sunset})}$$

Here $\alpha = 0.25$ and $d = 0.1$ have been set in the simulations. Note that in this definition T_{min} refers to the minimum temperature ahead in time. For the night time after midnight this is the minimum temperature at the day itself, but after sunset and before midnight this is the minimum temperature of the next day. Similarly, after midnight T_{sunset} refers to the temperature at sunset of the previous day, but before midnight it refers to the temperature at sunset at the day itself. The values for T_{sunset} are obtained from the daytime function $dot(t)$. The overall outdoor temperature function is defined as $\max(dot(t), not(t))$; note that after sunset and before sunrise the function $dot(t)$ is set at a large negative value, so that then always the $not(t)$ is taken in the max operator.

4.2. Modelling the Indoor Temperature

For the indoor temperature it is assumed that as long as the indoor temperature stays above the assumed goal temperature T_g natural cooling down takes place, and otherwise it stays at the goal

temperature. Therefore the indoor temperature at some time point $t + \Delta t$ is determined in two steps:
 (1) Determine how much cooling down takes place from t to $t + \Delta t$ when no heating is done:

$$T_{id}(t) - (\varepsilon/24C)*(T_{id}-T_{od})\Delta t$$

(2) The indoor temperature at $t + \Delta t$ is set on the maximum of the above cooled down value and the goal indoor temperature T_g
 So,

$$T_{id}(t + \Delta t) = \max(T_{id}(t) - (\varepsilon/24C)*(T_{id}(t) - T_{od}(t))\Delta t, T_g)$$

Here in the simulations $C = 4.6$ and $\varepsilon = 0.083$ have been used.

4.3. Modelling Energy Demand

From the indoor and outdoor temperatures the heating energy demand is determined. The energy demand $ed(t, t+\Delta t)$ for the interval from t to $t+\Delta t$ is composed of a part $\varepsilon dd(t) \Delta t$ that is for compensation of the energy loss, and (when needed) a part $C(T(t+\Delta t)-T(t))$ that is used to increase the indoor temperature (to get it at the level of the goal temperature). Based on this the overall energy demand $ed(t+\Delta t)$ up to time $t+\Delta t$ is modelled as:

$$\begin{aligned} ed(t+\Delta t) &= ed(t) + ed(t, t+\Delta t) \\ &= ed(t) + \varepsilon dd(t) \Delta t + C (T(t+\Delta t)-T(t)) \end{aligned}$$

4.4. Modelling Energy Usage and Cost

From the energy demand per time interval from t to $t+\Delta t$ the energy usage and cost can be determined when the seasonal performance factor $SPF(t)$ and the price $\pi_{el}(t)$ of electricity at t are available. Then energy usage and cost for the interval from t to $t+\Delta t$ are given by

$$\begin{aligned} eu(t, t+\Delta t) &= ed(t, t+\Delta t) / SPF(t) \\ ec(t, t+\Delta t) &= eu(t, t+\Delta t) \pi_{el}(t) \end{aligned}$$

Based on this the energy demand, usage and cost up to time t are modelled as

$$\begin{aligned} eu(t+\Delta t) &= eu(t) + eu(t, t+\Delta t) \\ &= eu(t) + ed(t, t+\Delta t) / SPF(t) \\ ec(t+\Delta t) &= ec(t) + ec(t, t+\Delta t) \\ &= ec(t) + eu(t, t+\Delta t) \pi_{el}(t) \end{aligned}$$

So a main issue remaining is to estimate $SPF(t)$ for any given time point for which the outdoor

temperature is given. To obtain a reasonable estimation of how for a given water temperature the performance factor depends on the outdoor temperature, theoretical analyses or lab experiments can be performed. However, such theoretical analyses are often not guaranteed to provide values that occur in realistic situations. A different route is to take empirical data from realistic contexts as a point of departure and make an approximation of them by a mathematical function (cf. Fig. 2).

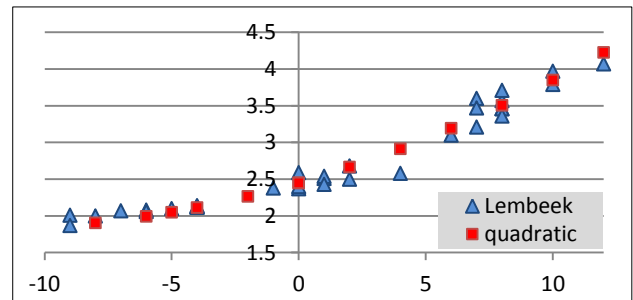


Fig. 2: Quadratic approximation of seasonal performance factors in relation to outdoor temperature compared to empirical data over 2012 for the site in Lembeek (water temperature 50 °C)

For example, in (Yang, Shi, and Dong, 2006, p. 2372) this was done based on the manufacturer's catalog data. However, a useful source of more realistic real world data can be found at the Website www.liveheat_pump.com. This approach was used in (Treur, 2013b) and is adopted here as well. In Fig. 2 a graph is shown with values from this Website for the average day temperature on the horizontal axis and the performance factor on the vertical axis (for water temperature in the heating system approximately 50°C).

More specifically, this has been done for the site at Lembeek, where the General Waterstage HT heat pump combination WH16/WOH16 is used. Moreover, in Fig. 1 a quadratic approximation of SPF is shown; this is assumed of the form (with $T_w = 50$):

$$SPF(T_{od}) = 7.45 - 0.1 * (T_w - T_{od}) + 0.004 T_{od}^2$$

Using the approximation (in Fig. 1, the seasonal performance factor SPF can be estimated on a daily basis throughout a year, when the day temperatures are given. This is one basic ingredient of the model for the heating agent used. A second ingredient of the model concerns how much energy for heating is

needed (energy demand), also depending on the outdoor temperature.

Fig. 2 shows the variables used in the heating agent model and the dependencies between them,

and Table 1 summarizes them. The right hand side in Fig. 2 describes how the performance factors are determined, and the left hand side describes how the energy demand is determined.

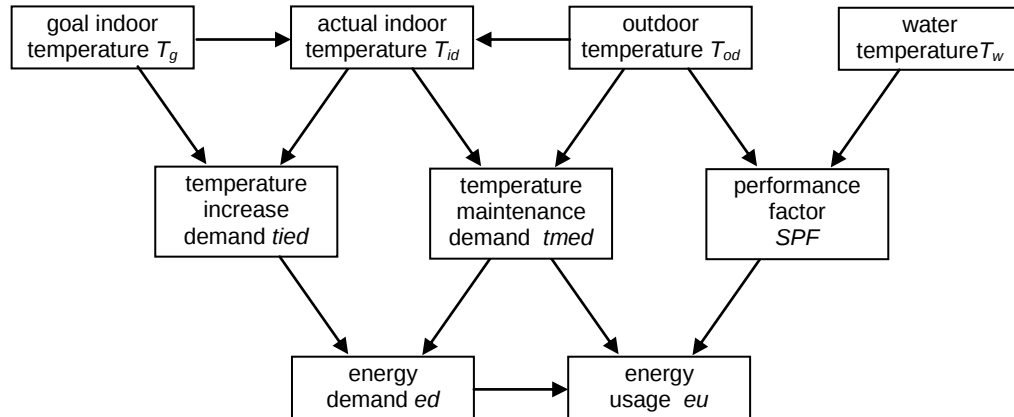


Fig. 3: Dependencies of the main variables relating to energy

5. Evaluation

Based on the computational model and a daily heating program that can be specified, a number of simulation experiments have been performed over the 24 hours of a specific day (with steps Δt of half an hour: 1/48 day), and this also for all days of all months in 2012 in which heating is needed. For some chosen daily heating programs detailed results have been obtained showing intraday dynamics, and these also have been aggregated into results for days of a month, for months in a year and for the whole year. For these simulations empirical data have been used on minimal and maximal day temperatures (of a given location in the Netherlands) for all days in 2012.

notation	description	unit
α	cooling down speed	0.25 /hour
d	minimal value margin	0.1 °C
ε	energy loss per degree day	4 kWh/°C day
C	capacity	4.6 kWh/°C

Tab. 2: Parameter values in the simulations

First an example is shown in Fig. 4 for the dynamics within one day, February 1, 2012, for a heating program with 18°C from 6 AM to 7 AM and 20°C from 2 PM to 9 PM and 14°C in the other time intervals (total day usage 42.6 kWh). The middle line shows the energy usage per half an hour. It can

be seen that the outdoor temperature on that day varied from -7.9 to -1.3. The indoor temperature shows the effect of the heating program. The energy usage shows two spikes of just above 10kWh in half an hour when the temperature is increased to the desired temperature at 6 AM and 2 PM. The temperature increase energy usage for this day is 23.5 kWh, so more than half of the overall day usage. Moreover, the temperature maintenance energy usage is about 1 to 2 kWh per half an hour, 19.1 kWh in total (less than half of the day usage); it is shown to be slightly higher when the outdoor temperature is lower.

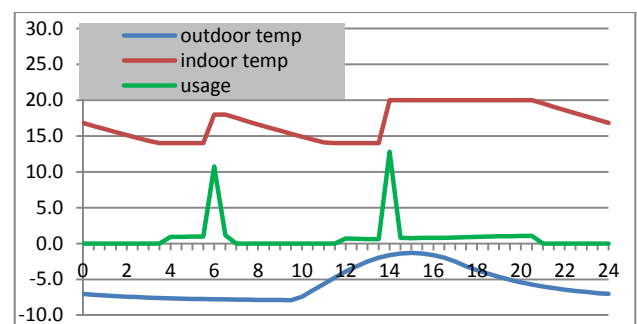


Fig. 4: Example simulation results: intraday dynamics

Next, an example is shown in Fig. 5 for the aggregated day usages in the month February for a daily heating program set on 18°C from 6 AM to 9 PM and 14°C in the night. As can be seen the first 12 days of this month were rather cold.

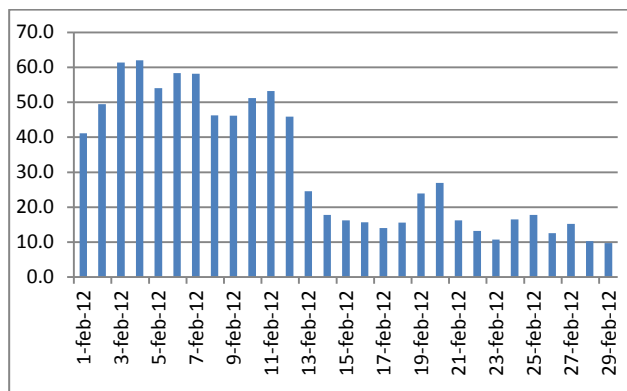


Fig. 5: Example simulation results: daily usages over one month

Two results of example simulations over a whole year are shown in Fig. 6. Here the energy usages over hours (and days) have been aggregated into usages per month (in kWh). The left hand bars (blue) show the results for a daily heating program that has goal temperature 16°C between 6 AM and 7 AM and goal temperature 18°C between 2 PM and 9 PM. For the other time intervals the goal temperature is 5°C, which for this case means no heating at all. The year usage for this scenario is 3257 kWh. The bars at the right hand side (red) show results for a heating program with goal temperature 18°C from 6 AM to 9 PM, and 5°C in the night. For this case the year usage is 3887 kWh.

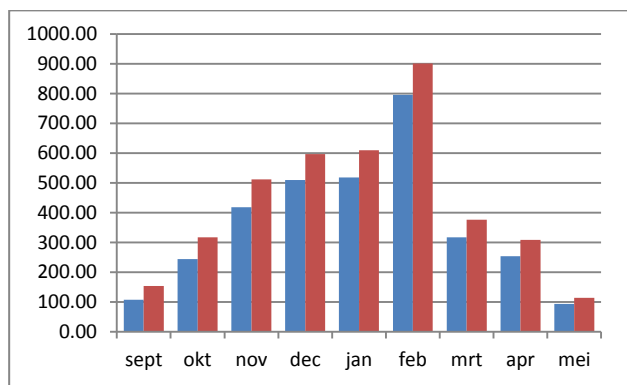


Fig. 6: Example simulation results: month usages for two different heating programs

For the second daily heating program indicated above it has been explored what the effect is of a higher temperature maintained at night: from 14°C to 18°C (in this latter case the temperature is kept constant 18°C for all 24 hours). The resulting year usages are shown in Fig. 7.

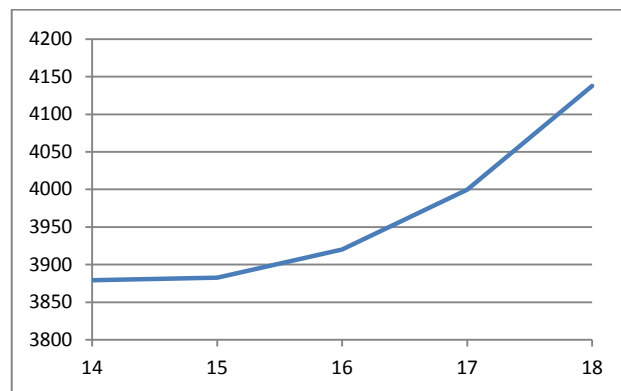


Fig. 7: Comparison of simulation results: year usages for different maintained indoor night temperatures

6. Discussion

The introduced model can be used to perform fine-grained intraday simulations of the energy usage for heating by a heat pump. As an example, it can be found out what difference it makes when the house is not heated in the night, or when the temperature is maintained at a lower level. As can be seen in Fig. 7 there is an advantage up to 250 kWh per year of the 4138 kWh overall in having lower temperatures in the night until 15°C. Lower than 15°C in the night does not come with additional advantages. So this provides an answer to the question in the introduction whether and to which extent it makes sense to have lower temperatures in the night. It makes sense but not lower than 16°C or 15°C. The advantage of a lower temperature can be savings of up to 6%.

7. Conclusion

In this paper it has been shown how a computational model can be used to estimate the energy usage of heating by an air-to-water heat pump for different daily heating programs. The model is particularly useful to find out what the effect on energy usage is of variations in a daily heating program. In other work the annual usage of a hybrid combination of a heat pump in combination with gas-based heating has been investigated (cf. Treur, 2013a). Moreover, it has been investigated how over a year the energy usage of a heat pump is covered by solar energy production based on a PV-system (cf. Treur, 2013b).

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