Pulse Compression to the Subphonon Lifetime Region by Half-Cycle Gain in Transient Stimulated Brillouin Scattering

Iavor Velchev, Dragomir Neshev, Wim Hogervorst, and Wim Ubachs

***Abstract*— A new approach to the transient pulse compression by stimulated Brillouin scattering (SBS) is presented. The theoretical analysis involving the time-dependent SBS gain in explicit form leads to a nonlinear system of partial differential equations, solved numerically by a generalization of the split-step method. It is shown theoretically and confirmed experimentally that the phonon lifetime is not always an appropriate parameter that determines the lower limit to the pulse duration in SBS compressors. A half-cycle gain regime is found for pulses shorter than the phonon lifetime. Hence, under proper conditions, pulses as short as half the acoustic period can be produced.**

***Index Terms*—Brillouin scattering, optical pulse compression, optical pulses.**

I. Introduction

The phenomenon of stimulated Brillouin scattering (SBS) in liquid or gaseous media is nowadays widely used as a tool to compress nanosecond laser pulses down to the subnanosecond region with remarkable conversion efficiency, theoretically reaching 100%. The first experimental result, some 20 years ago [1], inspired many attempts to develop a theory explaining the physical background of the process of pulse compression. Although most of the theories available are in good agreement with experimental results, they fail in treating the problem in the fully transient regime, where the compressed pulse is found to be shorter than the phonon lifetime of the medium. In this paper, we present a new analysis of the equations as well as a numerical method for modeling the compression in transient regime. A double compressor experiment is conducted in order to confirm the theoretical and numerical results.

II. Theory of Transient SBS

We consider the SBS process involving two classical optical fields E_1 (laser) and E_2 (Stokes) governed by Maxwell's equations, coupled through the process of electrostriction with an acoustic field $\bar{\rho}$, obeying the Navier–Stokes equation. Since our model is $1+1$ dimensional, these three fields are represented as scalar plane waves, possessing time and propagation coordinates only

$$E_1(z,t) = A_1(z,t)e^{i(k_1 z - \omega_1 t)} + c.c. \tag{1a}$$

$$E_2(z,t) = A_2(z,t)e^{i(-k_2 z - \omega_2 t)} + c.c. \tag{1b}$$

$$\bar{\rho}(z,t) = \rho_0 + \rho(z,t)e^{i(q_B z - \Omega_B t)} + c.c. \tag{1c}$$

In (1a)–(1c), the frequencies and the wavevectors satisfy energy $(\omega_1 = \omega_2 + \Omega_B)$ and momentum $(k_1 = q_B - k_2)$ conservation laws. Under the assumption of small Stokes shift $(\omega = \omega_1 \approx \omega_2)$, the acoustic frequency is given by $\Omega_B = 2nv\omega/c$, where v is the speed of hypersound in the medium. The slowly varying amplitude approximation is applied for both laser and Stokes fields. For the acoustic field, this approximation is not valid as long as its spectral width is usually only an order of magnitude smaller than the main frequency Ω_B. Following these remarks, we write the equations in the form

$$\frac{n}{c}\frac{\partial A_1}{\partial t} + \frac{\partial A_1}{\partial z} = i\frac{\gamma_e \omega}{2nc\rho_0}\rho A_2 \tag{2a}$$

$$\frac{n}{c}\frac{\partial A_2}{\partial t} - \frac{\partial A_2}{\partial z} = i\frac{\gamma_e \omega}{2nc\rho_0}\rho^* A_1 \tag{2b}$$

$$\frac{\partial^2 \rho}{\partial t^2} + (\Gamma_B - i2\Omega_B)\frac{\partial \rho}{\partial t} - i\Gamma_B\Omega_B\rho = \frac{\gamma_e q_B^2}{4\pi}A_1 A_2^* \tag{2c}$$

retaining the second-order time derivative in the Navier–Stokes equation (2c). The only restriction imposed on the acoustic field is the approximation of nonmoving phonons, which is well justified on a time scale of several hundred picoseconds [2]. In (2a)–(2c), n is the refractive index of the medium, c is the speed of light, γ_e is the electrostrictive constant, ρ_0 is the unperturbed density, and Γ_B is the Brillouin linewidth [2]. This set of coupled nonlinear differential equations is difficult to solve generally and analytical solutions have been found in specific situations only. The steady-state solution, for example, has been extensively investigated [2], [3]. A solution in the case of undepleted pump has also been found [4]. Despite all these attempts, no general approach to the problem has been proposed so far. In the following, we present an analysis of the system (2a)–(2c) under no additional approximations, revealing the physical background underlying the process of transient pulse compression by SBS.

We solve (2c) in the frequency domain, thus obtaining the spectrum $\tilde{\rho}(z,\Omega)$ of the acoustic field as a function of detuning

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(Ω) from the Stokes resonance

$$\tilde{\rho}(z,\Omega) = -\frac{\gamma_e q_B^2}{4\pi\Omega_B}\tilde{g}(\Omega)\cdot\widetilde{A_1 A_2}^* \tag{3a}$$

$$\tilde{g}(\Omega) = \frac{\Omega_B}{(\Omega-\Omega_B)^2 - \Omega_B^2 - i\Gamma_B(\Omega-\Omega_B)}. \tag{3b}$$

The function $\tilde{g}(\Omega)$ is the spectral gain profile of the SBS process possessing two resonances of widths $\Delta\Omega_{FWHM} = \Gamma_B$ at $\Omega = 0$ and $\Omega = 2\Omega_B$, corresponding to Stokes and anti-Stokes scattering, respectively.

Inverse Fourier transformation of (3a) and (3b) gives the exact solution of the Navier–Stokes equation

$$\rho(z,t) = -\frac{\gamma_e q_B^2}{4\pi\Omega_B} g(t)\otimes(A_1 A_2^*) \tag{4a}$$

$$g(t) = \begin{cases} 0, & t<0 \\ \dfrac{-\sqrt{2\pi}\,\Omega_B e^{-(\Gamma_B/2)t} e^{i\Omega_B t}\sin\left(\sqrt{\Omega_B^2 - \frac{\Gamma_B^2}{4}}\,t\right)}{\sqrt{\Omega_B^2 - \frac{\Gamma_B^2}{4}}}, & t\geq 0 \end{cases} \tag{4b}$$

where $\otimes$ denotes convolution. The fact that the gain function (4b) is zero for $t<0$ removes the integration over the *future* ($\tau>t$) of the optical fields in (4a)

$$\rho(z,t) = -\frac{\gamma_e q_B^2}{4\pi\Omega_B}\,\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t} g(t-\tau)A_1(z,\tau)A_2^*(z,\tau)\,d\tau. \tag{5}$$

This expression is a direct consequence from both the *locality* (phonon is generated at a fixed coordinate z only if at a certain moment t_0 both Stokes and laser fields are present) and *inertia* of the phonons [the acoustic field exists even after the optical fields have been removed and vanishes exponentially in time (4b)]. The latter determines the time scale of the acoustooptical interaction known as phonon lifetime ($\tau_p = 1/\Gamma_B$). It is usually regarded as a limiting factor to the pulse duration in SBS pulse compressors [6], [7]. It is important to note that (4a) and (4b) give general solution of the Navier–Stokes equation without any restriction to the time scale used. In the particular case of CW interaction, (4a) is transformed into the well-known steady-state solution where the convolution integral (5) is easy to solve. This results in an expression for the acoustic wave no longer dependent on time

$$\rho(z) = -\frac{\gamma_e q_B^2}{4\pi\Omega_B}A_1 A_2^*\left(-\frac{i}{\Gamma_B}\right). \tag{6}$$

Then the system (2a) and (2b) for the optical fields, written in terms of intensities ($I_i = (nc/2\pi)A_i A_i^*$), describes a pure gain (loss) process

$$\begin{aligned}\frac{dI_1}{dz} &= -g_B I_2 I_1 \\ \frac{dI_2}{dz} &= -g_B I_1 I_2\end{aligned} \tag{7}$$

where $g_B = (\gamma_e^2\omega^2)/(nc^3\upsilon\rho_0\Gamma_B)$ is the steady-state Brillouin gain. The system (7) is discussed widely in the literature, taking into account the depletion of the pump wave. It is found to describe the SBS process for CW interaction in optical fibers [5] as well as in the pulsed regime on a time scale much larger than the phonon lifetime τ_p in the medium.

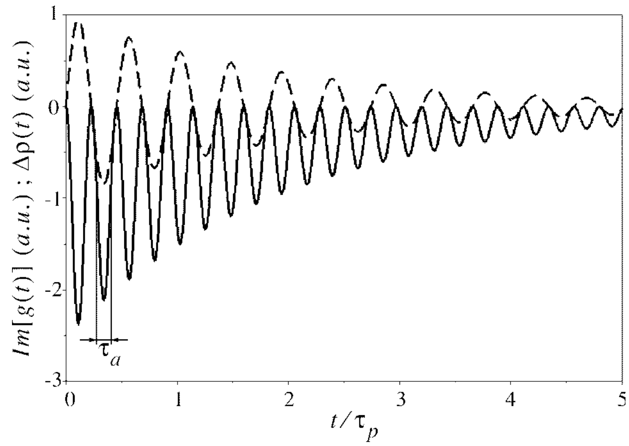

Fig. 1. Imaginary part of the gain function $g(t)$ (solid line) and the acoustic field $\Delta\rho(t)$ (dashed line) generated by a short δ-like pulse in water for $\lambda = 532$ nm. Note that the ratio τ_p/τ_a is a material property.

Another important feature of the steady-state solution appears in (6). The fact that the convolution integral $(-i/\Gamma_B)$ is both *imaginary* and *negative* leads to the pure gain (loss) system (7). In the opposite case, when a short δ-like pulse is propagating through a Brillouin medium, the convolution integral is proportional to the gain function $g(t)$ itself. The analysis of its imaginary part shows (Fig. 1) that it is always *negative*, reaching maximum absolute values when the density deviation from ρ_0 has a maximum and vanishes exponentially in time. This result is important in showing two time scales for the interaction of light pulses with a Brillouin active medium: first, the decay time of the acoustic field $\tau_p = 1/\Gamma_B$, and second, the duration of one oscillation of the gain function equal to $\tau_a = \pi/(2\Omega_B)$. For pulses longer than τ_p, the interaction is limited by the decay time of the acoustic field. This effect was observed in all attempts to compress pulses with a duration of several nanoseconds in liquid media (with phonon lifetimes on the order of several hundred picoseconds) [6], [7]. However, if the incident pulse is shorter and its duration is on the order of τ_a, it cannot experience gain due to a lack of time to build up an acoustic field. Consequently, the real physical limit to the pulse duration in a compressor setup is not the phonon lifetime τ_p, but the acoustical half-cycle duration τ_a. A conclusion along these lines was first drawn by Hon [1] without any mathematical derivation. Later, this problem was addressed in [8]–[10] on a qualitative level as well.

The system (2a)–(2c) can be rewritten in the form

$$\frac{\partial}{\partial t}A_1(z,t) + \frac{c}{n}\,\frac{\partial}{\partial z}A_1(z,t) = -i\alpha\frac{c}{n}\varrho(z,t)A_2(z,t) \tag{8a}$$

$$\frac{\partial}{\partial t}A_2(z,t) - \frac{c}{n}\,\frac{\partial}{\partial z}A_2(z,t) = -i\alpha\frac{c}{n}\varrho^*(z,t)A_1(z,t) \tag{8b}$$

$$\varrho(z,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t} g(t-\tau)A_1(z,\tau)A_2^*(z,\tau)\,d\tau \tag{8c}$$

where $\alpha = g_B\Gamma_B/2$ is the coupling constant. It was discussed in detail in [8] that under transient conditions the physically important parameter is the product $g_B\Gamma_B$ instead of the steady-state gain coefficient g_B alone. In a transient regime, the integral (8c) plays an important role in the model described. It can be considered as a *memory* of the system, stored into the acoustic field $\varrho(z,t)$, which depends strongly on the past values of the product $A_1(z,\tau)A_2^*(z,\tau)$ for times $\tau \in (t-\tau_p, t)$. In the nontransient regime, τ_p is much shorter than the pulse durations and the acoustic field depends on the present values of the product $A_1(z,t)A_2^*(z,t)$ only, thus simplifying the problem to a system which is easy to model without computing the integral (8c) [7].

III. Numerical Model

We solve the system (8a)–(8c) numerically by a generalization of the split-step method [5] usually used for modeling pulse propagation in optical fibers. We extended the applicability of this method to our case of two nonlinearly coupled [through the integral (8c)] partial differential equations. As long as the phonons do not propagate and their lifetime cannot be neglected with respect to the pulse durations, it is obvious that the fields evolve in time only, whereas in systems with a short response time (Kerr nonlinearity) the evolution in time can be replaced by evolution in space (along the z coordinate). Consequently, the system (8a) and (8b) describes the evolution of the spatial distribution (along the z axis) of the fields in time.

By introducing a *propagation operator* $\hat{P} = (c/n)(\partial/\partial z)$ and a *nonlinearity operator* $\hat{N} = -i\alpha(c/n)\varrho\cdot$, (8a) and (8b) can be rewritten in vector form

$$\frac{\partial}{\partial t}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \left(\begin{bmatrix} 0 & \hat{N} \\ -\hat{N}^* & 0 \end{bmatrix} + \begin{bmatrix} -\hat{P} & 0 \\ 0 & \hat{P} \end{bmatrix}\right)\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}. \tag{9}$$

The split-step method gives an approximate solution of (9) by assuming that, for a small time increment Δt, the propagation and the nonlinearity act independently. Under this assumption, the evolution of the optical fields in time is given by

$$\begin{pmatrix} A_1(z,t+\Delta t) \\ A_2(z,t+\Delta t) \end{pmatrix} = \exp\left\{\begin{bmatrix} 0 & \hat{N} \\ -\hat{N}^* & 0 \end{bmatrix}\Delta t\right\} \times \exp\left\{\begin{bmatrix} -\hat{P} & 0 \\ 0 & \hat{P} \end{bmatrix}\Delta t\right\}\cdot\begin{pmatrix} A_1(z,t) \\ A_2(z,t) \end{pmatrix}. \tag{10}$$

In (10), the propagation exponent operator acts first, followed by the nonlinearity exponent operator. The fact that the operators $\hat{P}$ and $\hat{N}$ are noncommuting is the predominant error source in the split-step method, limiting the accuracy to second-order in step size Δt [5]. It is easy to prove that the propagation exponent operator causes a shift $z \pm c\Delta t/n$ in the spatial part of the fields, where "+" is for the Stokes field and "−" is for the laser field. The nonlinearity exponent operator can be presented in explicit form using trigonometric functions. Inserted into (10), this results in

$$\begin{pmatrix} A_1(z,t+\Delta t) \\ A_2(z,t+\Delta t) \end{pmatrix} = \begin{bmatrix} \cos(\theta) & -i\dfrac{\varrho}{|\varrho|}\sin(\theta) \\ -i\dfrac{\varrho^*}{|\varrho|}\sin(\theta) & \cos(\theta) \end{bmatrix} \times \begin{pmatrix} A_1\left(z - \dfrac{c}{n}\Delta t, t\right) \\ A_2\left(z + \dfrac{c}{n}\Delta t, t\right) \end{pmatrix} \tag{11}$$

where $\theta = \alpha|\varrho|(c/n)\Delta t$. The implementation of (11) gives the time evolution of both pump and Stokes fields in each step. The method is applicable only if the spatial grid spacing Δz is equal to $c\Delta t/n$, allowing for the introduction the spatial shifts $z \pm c\Delta t/n$ without additional calculations. Reducing the step size, the memory consumption increases, thus limiting the speed and accuracy of the calculation. This constraint could be eliminated by using fast Fourier transformations [5] for calculation of the action of the propagation exponent operator in (10). In our case, the computer memory available was enough to obtain good accuracy by lowering the step size. We used (11) directly to model the process of SBS pulse compression. In each step, the accuracy was monitored by the conservation of the number of photons. Another accuracy check necessary when using the split-step method is recalculation with two-fold reduced step size. Both checks showed good accuracy with moderate memory consumption and speed of calculation.

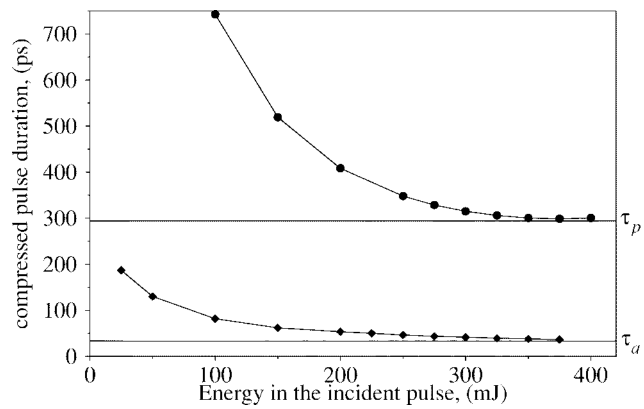

Fig. 2. Calculated compressed pulse duration *versus* energy in the incident pulse for a 10-mm beam diameter. Circles: input pulse of 5 ns; diamonds: input pulse of 600-ps duration.

IV. Results and Discussion

The experimental situation modeled is a generator-amplifier setup [7], where the laser beam is focused by a lens in water as a Brillouin active medium. The Stokes signal generated in the focal region propagates backward, depleting the remainder of the pump pulse on its way [1], [7], [8]. As a result of this interaction, the front edge of the Stokes pulse is amplified only, leading to pulse compression. In our model, a Gaussian beam/pulse at 532 nm of 5-ns (FWHM) duration with up to 300 mJ/pulse is focused by a 10-cm lens in water. The Brillouin shift is $\Omega_B/2\pi = 7.42$ GHz and the Brillouin linewidth is $\Gamma_B/2\pi = 539$ MHz extracted from [2] after

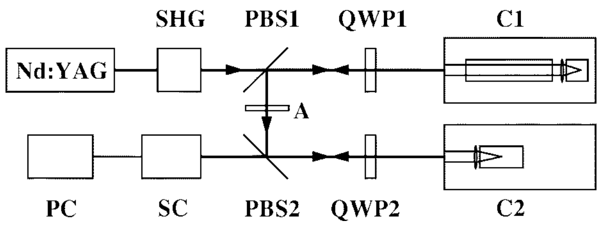

Fig. 3. Experimental setup of double compressor. SHG: second-harmonic generation stage; PBS: polarizing beam splitter; QWP: quarter-wave plate; A: attenuator; C: Brillouin compressor; SC: streak camera; PC: personal computer.

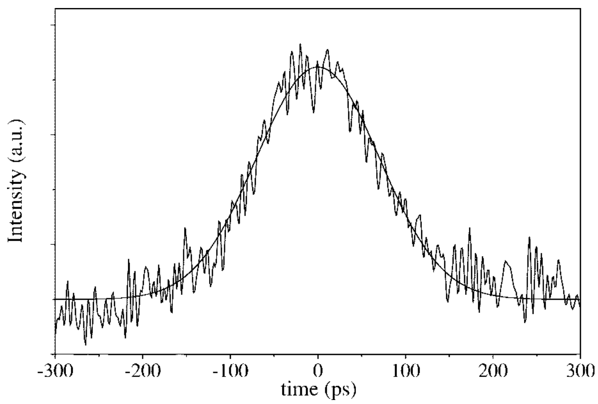

Fig. 4. Streak camera trace of the secondary compressed pulse in water. The measured pulse duration (FWHM) is 160(10) ps much shorter than the phonon lifetime τ_p.

wavelength correction for $\lambda = 532$ nm. In Fig. 2, a graph of the calculated compressed pulse duration *versus* energy in the input pulse is presented. Circles and diamonds correspond to input pulse durations of 5 ns and 600 ps, respectively. It is clearly seen that, in the case of a 5-ns input pulse, the limit to the pulse compression is set by the phonon lifetime τ_p. This result is in agreement with all available experimental results on compression of long $(\tau \gg \tau_p)$ pulses [6], [7]. The situation is different for a 600-ps input pulse, which enters directly the transient regime. In this case, our model predicts compressed pulses much shorter than τ_p. As discussed above, the duration τ_a of one oscillation of the gain function $g(t)$ is assumed to be the limit to transient pulse compression. It can be seen from Fig. 2 that, increasing the energy in the input pulse, the compressed pulse duration is indeed limited by τ_a.

The behavior of the conversion efficiency in a fully transient regime is difficult to predict on a time scale shorter than τ_p. Equations (4a) and (4b) suggest that pulses shorter than τ_a experience reduced gain, and, therefore, the conversion efficiency is considerably lower. Between the two limits, our numerical experiment showed no significant deviation from 98% maximum efficiency.

V. Experiment

To compress a pulse shorter than the phonon lifetime, we performed a double compression experiment. The setup is shown in Fig. 3. The output of the injection-seeded Nd:YAG laser was frequency-doubled ($\lambda = 532$ nm) and compressed to 300 ps (FWHM) with 5 mJ energy in the pulse. Water was used as a Brillouin medium in a second compressor stage (C2), where the pulse was further compressed. The output pulse duration was measured with a streak camera read by computer. The uncertainty in this single-shot measurement is determined by the statistical error in the fitting procedure. In Fig. 4, a streak camera trace of the two-fold compressed pulse is presented, measured to be 160(10) ps (FWHM). This value is much shorter than the phonon lifetime in water $\tau_p = 295$ ps [2], thus experimentally proving our prediction that the limit to the pulse compression in this case is not set by the phonon lifetime. Even shorter pulses could, in principle, be achieved at higher pulse energies down to the theoretical limit $\tau_a = 34$ ps for water at 532 nm. The true experimental limit, however, is determined by the competition with Raman scattering and optical breakdown. The first one could be eliminated by choosing a low Raman gain liquid or an atomic gas as a Brillouin medium. The optical breakdown limitation we overcome by filtering the liquid down to 200-nm particle size, using pulses with smooth temporal profile and limiting (by the attenuator A) the energy in the second compressor stage (C2) to 5 mJ/pulse.

VI. Conclusions

We have theoretically analyzed and numerically modeled the pulse compression by stimulated Brillouin scattering in the fully transient regime. By introducing a time-dependent gain, explicitly presented by (4b), we found a regime where the pulses are compressed in a half-cycle time τ_a. In a double compression experiment, we demonstrated the accessibility of the time region below τ_p when the initial pulse duration is on the order of the phonon lifetime.

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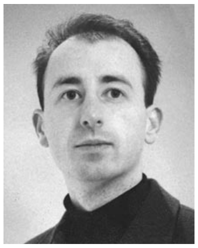

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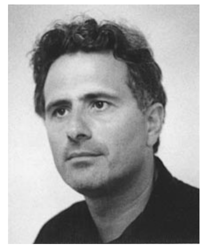

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