There is no largest prime number

Euclid of Alexandria
euclid@alexandria.edu

27th International Symposium of Prime Numbers

Outline

Outline

Outline

What are prime numbers?

Definition

A prime number is a number that has exactly two divisors

What are prime numbers?

Definition

A prime number is a number that has exactly two divisors

Example

- 2 is prime (two divisors: 1 and 2)

What are prime numbers?

Definition

A prime number is a number that has exactly two divisors

Example

- 2 is prime (two divisors: 1 and 2)
- 3 is prime (two divisors: 1 and 3)

What are prime numbers?

Definition

A prime number is a number that has exactly two divisors

Example

- 2 is prime (two divisors: 1 and 2)
- 3 is prime (two divisors: 1 and 3)
- 4 is not prime (three divisors: 1, 2 and 4)

An algorithm for finding prime numbers

```
int main(void)
{
  std::vector<bool> is_prime(100,true);
  for(int i = 2; i < 100; i++)









  return 0;
}
```

An algorithm for finding prime numbers

```
int main(void)
{
  std::vector<bool> is_prime(100,true);
  for(int i = 2; i < 100; i++)
    if(is_prime[i])
    {



    }
  return 0;
}
```

An algorithm for finding prime numbers

```
int main(void)
{
  std::vector<bool> is_prime(100,true);
  for(int i = 2; i < 100; i++)
    if(is_prime[i])
    {
      std::cout << i << " ";
      for(int j = i; j < 100;
        is_prime[j] = false, j+=i);
    }
  return 0;
}
```

An algorithm for finding prime numbers

```
int main(void)
{
  std::vector<bool> is_prime(100,true);
  for(int i = 2; i < 100; i++)
    if(is_prime[i])
    {
      std::cout << i << " ";
      for(int j = i; j < 100;
        is_prime[j] = false, j+=i);
    }
  return 0;
}
```

Note the use of `std::`

There is no largest prime number

The proof uses *reductio ad absurdum*

Theorem

There is no largest prime number

Proof.

1. Suppose p were the largest prime number

4. Thus $q + 1$ is also prime and greater than p □

There is no largest prime number

The proof uses *reductio ad absurdum*

Theorem

There is no largest prime number

Proof.

1. Suppose p were the largest prime number
2. Let q be the product of the first p numbers

4. Thus $q + 1$ is also prime and greater than p □

There is no largest prime number

The proof uses *reductio ad absurdum*

Theorem

There is no largest prime number

Proof.

1. Suppose p were the largest prime number
2. Let q be the product of the first p numbers
3. Then $q + 1$ is not divisible by any of them
4. Thus $q + 1$ is also prime and greater than p □

There is no largest prime number

The proof uses *reductio ad absurdum*

Theorem

There is no largest prime number

Proof.

1. Suppose p were the largest prime number
2. Let q be the product of the first p numbers
3. Then $q + 1$ is not divisible by any of them
4. Thus $q + 1$ is also prime and greater than p □

The proof used *reductio ad absurdum*

What's still to do?

Answered questions

How many primes are there?

What's still to do?

Answered questions

How many primes are there?

Open questions

Is every even number the sum of two primes? [1]

References


[Goldbach, 1742] Christian Goldbach
A problem we should try to solve before the ISPN ’43 deadline
Letter to Leonhard Euler, 1742